Exponentials and Logarithms- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

2.

Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

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3.

The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

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4.

The value of a car, £V, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925e^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

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5.

A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\frac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

6.

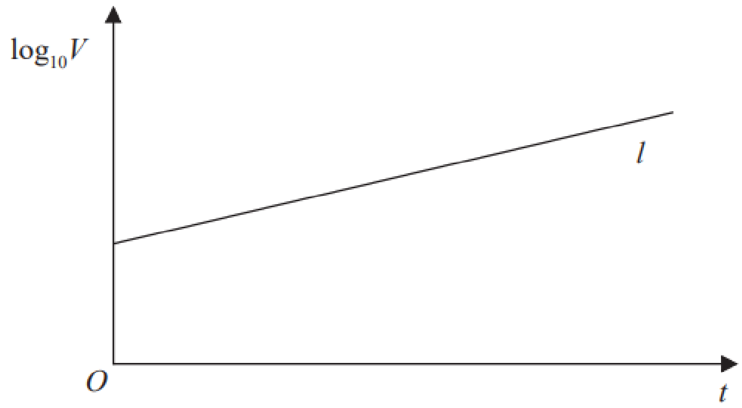

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

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7.

7. (i) $$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

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8.

8 (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2,$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0,$$

giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

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9.

7. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}.$$

(6)

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10.

7. (i) Find the exact value of x for which

$$\log_2 (2x) = \log_2 (5x + 4) - 3.$$

(4)

(ii) Given that

$$\log_a y + 3 \log_a 2 = 5,$$

express y in terms of a.

Give your answer in its simplest form.

(3)

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11.

6. Given that $2 \log_2 (x + 15) - \log_2 x = 6$,

(a) show that $x^2 - 34x + 225 = 0$.

(5)

(b) Hence, or otherwise, solve the equation $2 \log_2 (x + 15) - \log_2 x = 6$.

(2)

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12.

2. Find the values of x such that

$$2 \log_3 x - \log_3(x - 2) = 2$$

(5)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

13.

4. Given that $y = 3x^2$,

(*a*) show that $\log_3 y = 1 + 2\log_3 x$.

(3)

(*b*) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3 (28x - 9).$$

(3)

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14.

3. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(*a*) $5^x = 10$,

(2)

(*b*) $\log_3 (x - 2) = -1$.

(2)

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15.

7. (*a*) Given that

$$2\log_3 (x - 5) - \log_3 (2x - 13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(*b*) Hence, or otherwise, solve $2\log_3 (x - 5) - \log_3 (2x - 13) = 1$.

(2)

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16.

5. (*a*) Find the positive value of x such that

$$\log_x 64 = 2.$$

(2)

(*b*) Solve for x

$$\log_2 (11 - 6x) = 2 \log_2 (x - 1) + 3.$$

(6)

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17.

2. Find the exact solutions, in their simplest form, to the equations

(*a*) $e^{3x-9} = 8$

(3)

(*b*) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

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18.

2. Find the exact solutions, in their simplest form, to the equations

(*a*) $2 \ln (2x + 1) - 10 = 0$

(2)

(*b*) $3^x e^{4x} = e^7$

(4)

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19.

6. Find algebraically the exact solutions to the equations

(*a*) $\ln (4 - 2x) + \ln (9 - 3x) = 2 \ln (x + 1), \quad -1 < x < 2,$

(5)

(*b*) $2^x e^{3x+1} = 10.$

Give your answer to (*b*) in the form $\dfrac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

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20.

8. (*a*) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}.$$

(3)

Given that

$$\ln(2x^2+9x-5)=1+\ln(x^2+2x-15), \quad x\neq -5,$$

(*b*) find x in terms of e.

(4)

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21.

9. (i) Find the exact solutions to the equations

(*a*) $\ln(3x-7)=5$,

(3)

(*b*) $3^x e^{7x+2}=15$.

(5)

(ii) The functions f and g are defined by

$$f(x)=e^{2x}+3, \qquad x\in\mathbb{R},$$

$$g(x)=\ln(x-1), \qquad x\in\mathbb{R}, \quad x>1.$$

(*a*) Find f^{-1} and state its domain.

(4)

(*b*) Find fg and state its range.

(3)